

Chapter 4

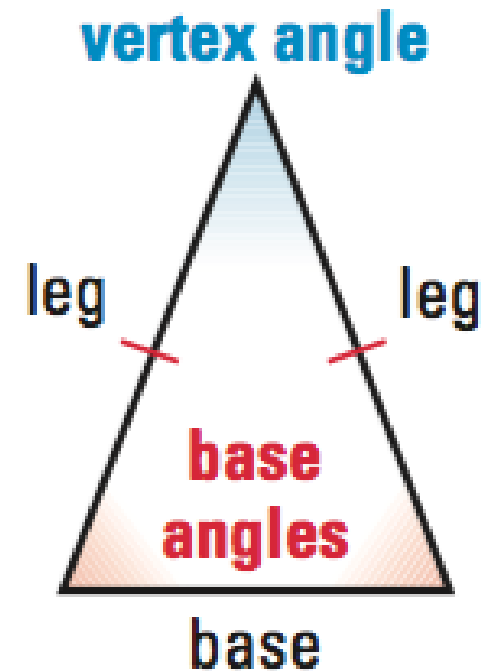
Congruent Triangles

Section 6

Isosceles, Equilateral, and Right Triangles

GOAL 1: Using Properties of Isosceles Triangles

In Lesson 4.1, you learned that a triangle is isosceles if it has at least two congruent sides. If it has exactly two congruent sides, then they are the legs of the triangle and the noncongruent side is the base. The two angles adjacent to the base are the **base angles**. The angle opposite the base is the **vertex angle**.

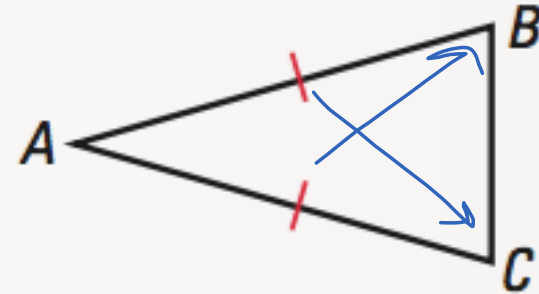


THEOREMS

THEOREM 4.6 *Base Angles Theorem*

If two sides of a triangle are congruent, then the angles opposite them are congruent.

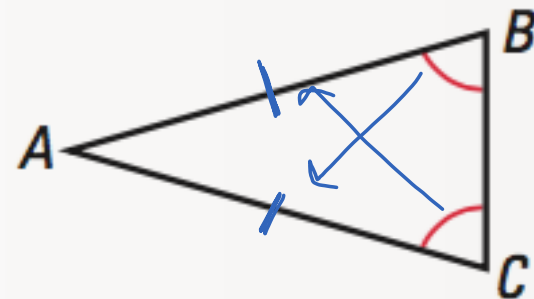
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



THEOREM 4.7 *Converse of the Base Angles Theorem*

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

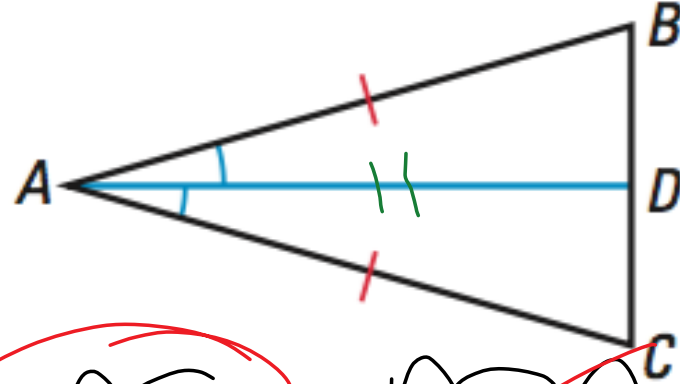


Example 1: Proof of the Base Angles Theorem

Use the diagram of $\triangle ABC$ to prove the Base Angles Theorem.

GIVEN ► $\triangle ABC, \overline{AB} \cong \overline{AC}$

PROVE ► $\angle B \cong \angle C$



~~SSS~~ SAS ~~ASA~~ ~~AAS~~

Statements

- 1) Tri. ABC; $\overline{AB} \cong \overline{AC}$
- 2) $\angle BAD \cong \angle CAD$
- 3) $\overline{AD} \cong \overline{AD}$
- 4) Tri. ABD \cong Tri. ACD
- 5) $\angle B \cong \angle C$

Reasons

- 1) Given
- 2) by construction (blue line)
- 3) reflexive (O.S.)
- 4) SAS
- 5) CPCTC

Recall that an equilateral triangle is a special type of isosceles triangle. The corollaries below state that a triangle is equilateral if and only if it is equiangular.

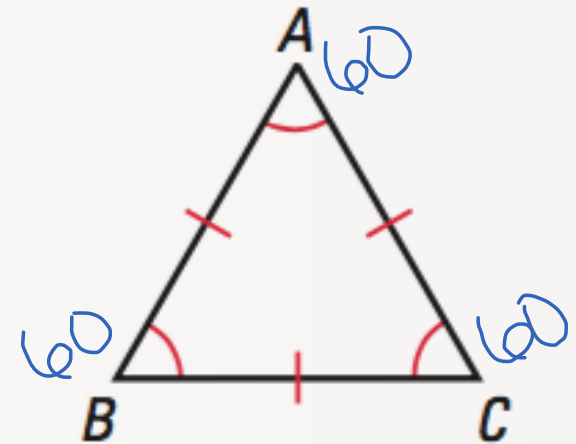
COROLLARIES

COROLLARY TO THEOREM 4.6

If a triangle is equilateral, then it is equiangular.

COROLLARY TO THEOREM 4.7

If a triangle is equiangular, then it is equilateral.

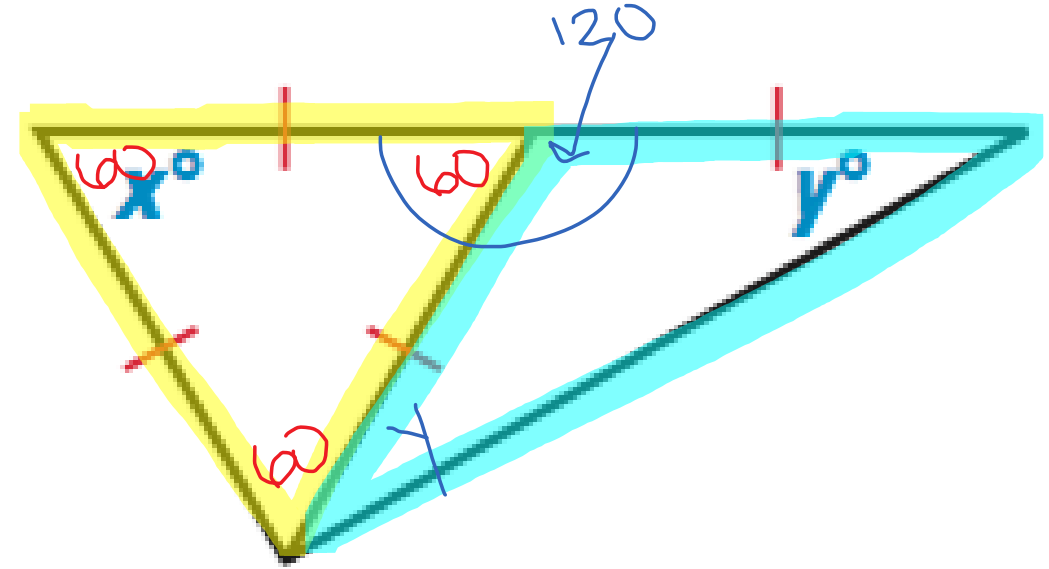


Example 2: Using Equilateral and Isosceles Triangles

a) Find the value of x .

equilateral \rightarrow equiangular

$$x = 60$$



a) Find the value of y .

$$y + y + 120 = 180$$

$$2y + 120 = 180$$

$$2y = 60$$

$$y = 30$$

GOAL 2: Using Properties of Right Triangles

You have learned four ways to prove that triangles are congruent.

- Side-Side-Side (SSS) Congruence Postulate (p. 212)
- Side-Angle-Side (SAS) Congruence Postulate (p. 213)
- Angle-Side-Angle (ASA) Congruence Postulate (p. 220)
- Angle-Angle-Side (AAS) Congruence Theorem (p. 220)

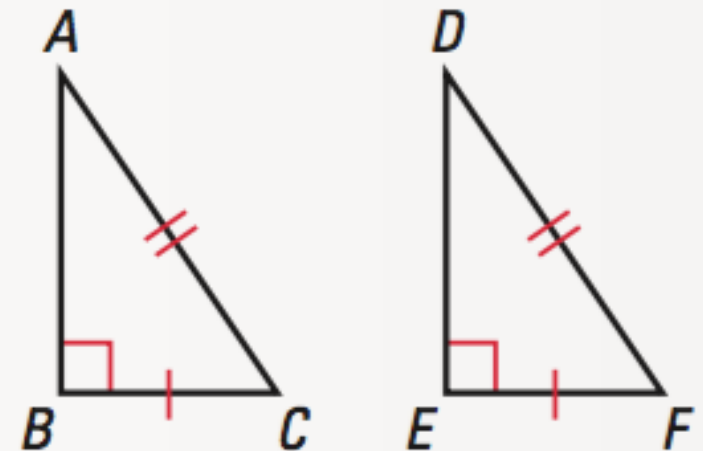
The Hypotenuse-Leg Congruence Theorem below can be used to prove that two right triangles are congruent.

THEOREM

THEOREM 4.8 *Hypotenuse-Leg (HL) Congruence Theorem*

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{BC} \cong \overline{EF}$ and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

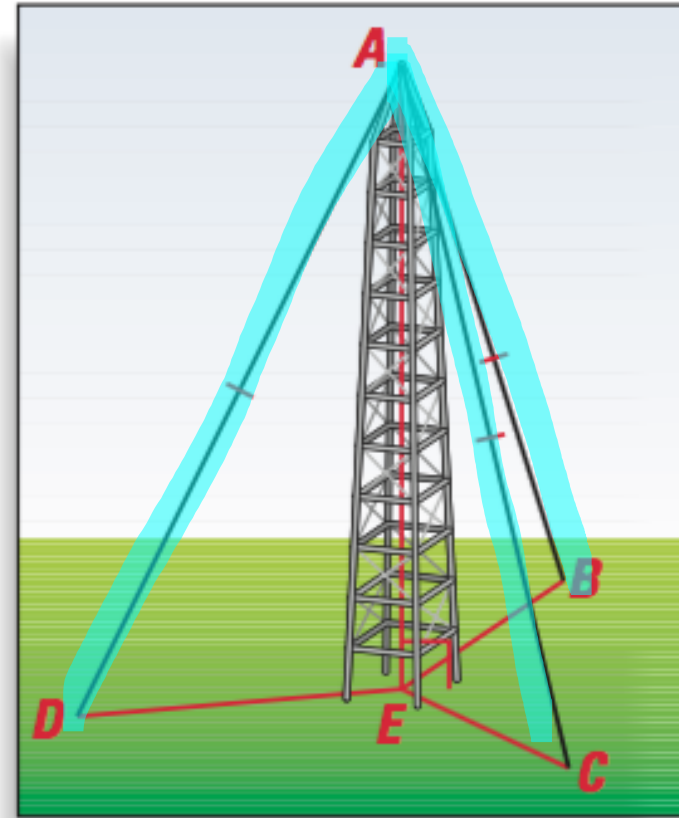



Example 3: Proving Right Triangles Congruent

The television antenna is perpendicular to the plane containing the points B , C , D , and E . Each of the stays running from the top of the antenna to B , C , and D uses the same length of cable. Prove that $\triangle AEB$, $\triangle AEC$, and $\triangle AED$ are congruent.

GIVEN ► $\overline{AE} \perp \overline{EB}$, $\overline{AE} \perp \overline{EC}$,
 $\overline{AE} \perp \overline{ED}$, $\overline{AB} \cong \overline{AC} \cong \overline{AD}$



PROVE ► $\triangle AEB \cong \triangle AEC \cong \triangle AED$



GIVEN ► $\overline{AE} \perp \overline{EB}, \overline{AE} \perp \overline{EC},$
 $\overline{AE} \perp \overline{ED}, \overline{AB} \cong \overline{AC} \cong \overline{AD}$ 

PROVE ► $\triangle AEB \cong \triangle AEC \cong \triangle AED$

Statements

- 1) 
- 2) $\angle AEB, \angle AEC, \angle AED$ are right \angle s
- 3) Tri. AEB, Tri. AEC, Tri. AED are right tri.
- 4) $\overline{AE} \cong \overline{AE} \cong \overline{AE}$ 
- 5) Tri. AEB \cong Tri. AEC \cong Tri. AED

Reasons

- 1) Given
- 2) Def. of perp. lines
- 3) Def. of right tri.
- 4) reflexive (O.S.)
- 5) HL

EXIT SLIP